

2. Trigonometric Functions

1. If A(x, y) is any point on the terminal arm OQ such that $OA = r = \sqrt{x^2 + y^2}$ and $\angle POQ = q$ then:

$$\sin q = \frac{y}{r}$$

$$\cos q = \frac{x}{r}$$

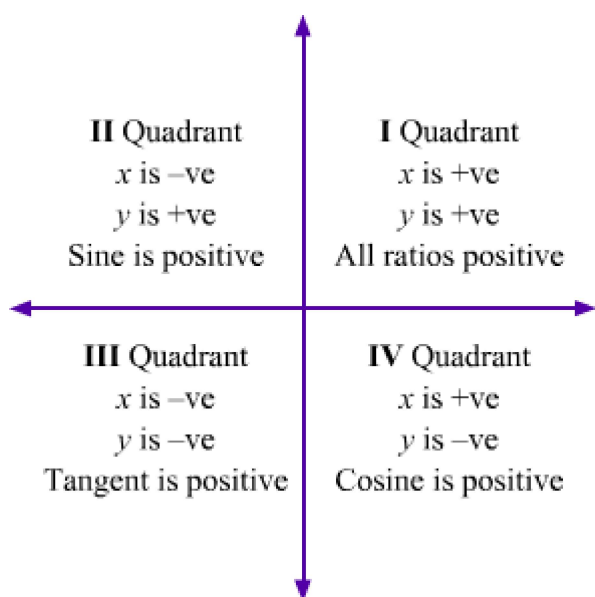
$$\tan q = \frac{y}{x}, \text{ where } x \neq 0$$

$$\operatorname{cosec} q = \frac{r}{y}, \text{ where } y \neq 0$$

$$\sec q = \frac{r}{x}, \text{ where } x \neq 0$$

$$\cot q = \frac{x}{y}, \text{ where } y \neq 0$$

2. The signs of various trigonometric ratios in different quadrants are as follows:



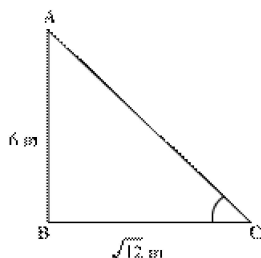
- Trigonometric Ratios of some specific angles

q	0	30°	45°	60°	90°
$\sin q$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos q$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan q$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} q$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec q$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot q$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 1:

$\triangle ABC$ is right-angled at B and $AB = 6$ m, $BC = \sqrt{12}$ m. Find the measure of $\angle A$ and $\angle C$.

Solution:



$$AB = 6 \text{ m,}$$

$$BC = \sqrt{12} \text{ m} = 2\sqrt{3} \text{ m}$$

$$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan C = \tan 60^\circ \quad \left[\because \tan 60^\circ = \sqrt{3} \right]$$

$$\Rightarrow \angle C = 60^\circ$$

$$\therefore \angle A = 180^\circ - (90 + 60) = 30^\circ$$

Example 2:

Evaluate the expression

$$4(\cos^3 60^\circ - \sin^3 30^\circ) + 3(\sin 30^\circ - \cos 60^\circ)$$

Solution:

$$4(\cos^3 60^\circ - \sin^3 30^\circ) + 3(\sin 30^\circ - \cos 60^\circ)$$

$$= 4 \left[\left(\frac{1}{2} \right)^3 - \left(\frac{1}{2} \right)^3 \right] + 3 \left(\frac{1}{2} - \frac{1}{2} \right)$$

$$= 4 \times 0 + 3 \times 0 = 0 + 0 = 0$$

- **Trigonometric Identities**

1. $\cos^2 A + \sin^2 A = 1$

2. $1 + \tan^2 A = \sec^2 A$

3. $1 + \cot^2 A = \operatorname{cosec}^2 A$

Example:

If $\cos \theta = \frac{5}{7}$, find the value of $\cot \theta + \operatorname{cosec} \theta$

Solution:

We have, $\cos \theta = \frac{5}{7}$

Now, $\sin^2 \theta + \cos^2 \theta = 1$

$$\therefore \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \left(\frac{5}{7}\right)^2}$$

$$= \sqrt{\frac{49-25}{49}} = \frac{2\sqrt{6}}{7}$$

$$\therefore \operatorname{cosec} \theta = \frac{7}{2\sqrt{6}}$$

Also, $\cot \theta = \frac{\cos \theta}{\sin \theta}$

$$= \frac{\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = \frac{5}{2\sqrt{6}}$$

$$\therefore \cot \theta + \operatorname{cosec} \theta = \frac{5}{2\sqrt{6}} + \frac{7}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \sqrt{6}$$

1. For any angle θ , we have

(i) $\sin(-\theta) = -\sin \theta$

(ii) $\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$

(iii) $\cos(-\theta) = \cos \theta$

(iv) $\sec(-\theta) = \sec \theta$

(v) $\tan(-\theta) = -\tan \theta$

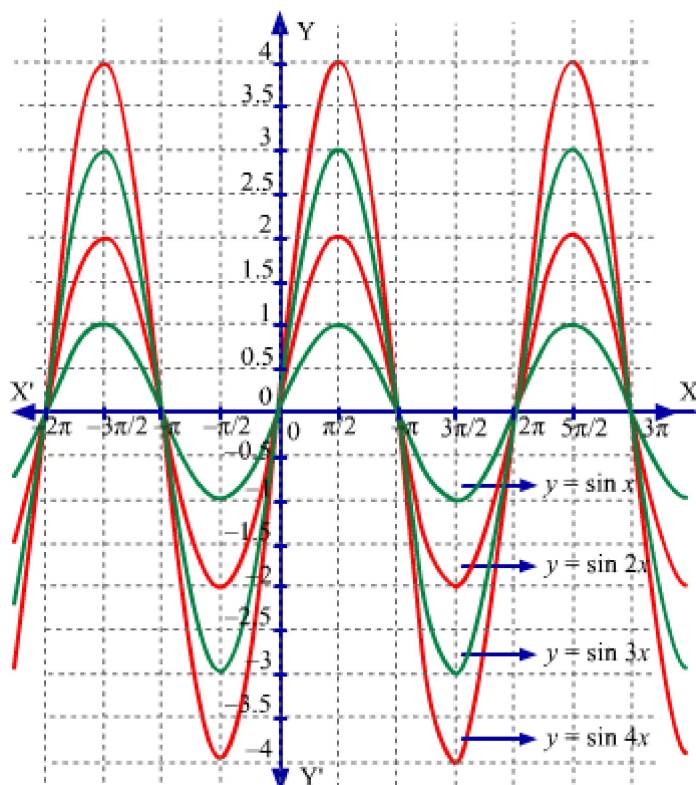
(vi) $\cot(-\theta) = -\cot \theta$

- **Domain and Range of trigonometric functions:**

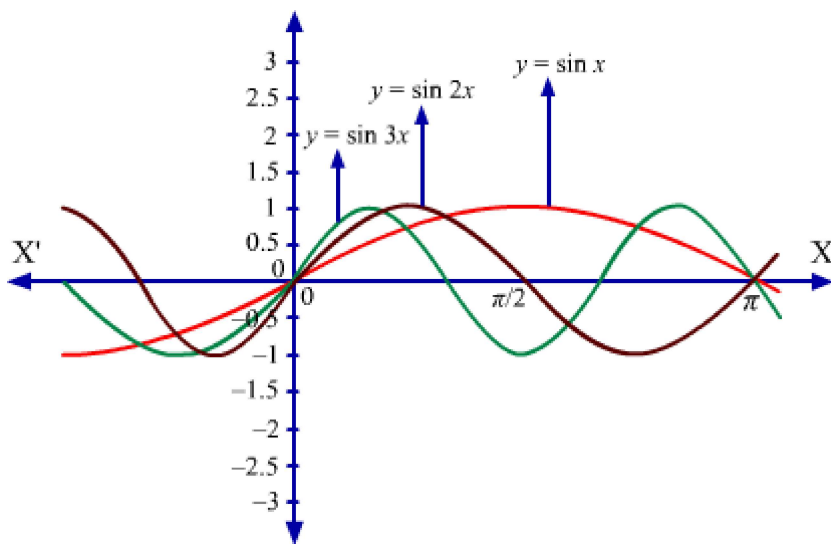
Trigonometric function	Domain	Range
$\sin x$	$\mathbf{R}$	$[-1, 1]$
$\cos x$	$\mathbf{R}$	$[-1, 1]$
$\tan x$	$\mathbf{R} - \left\{ x : x = \frac{(2n+1)\pi}{2}, n \in \mathbf{Z} \right\}$	$\mathbf{R}$
$\cot x$	$\mathbf{R} - \{ x : x = n\pi, n \in \mathbf{Z} \}$	$\mathbf{R}$
$\sec x$	$\mathbf{R} - \left\{ x : x = \frac{(2n+1)\pi}{2}, n \in \mathbf{Z} \right\}$	$\mathbf{R} - [-1, 1]$
$\operatorname{cosec} x$	$\mathbf{R} - \{ x : x = n\pi, n \in \mathbf{Z} \}$	$\mathbf{R} - [-1, 1]$

Graphs of Transformed Trigonometric Functions:

- Graphs of functions of the type $y = a \sin x$:

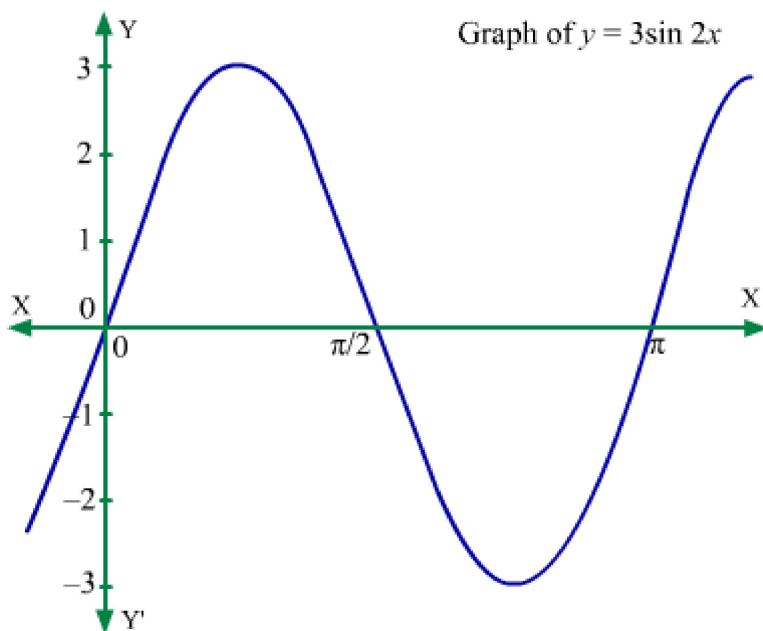


- Graphs of functions of the type $y = \sin bx$:

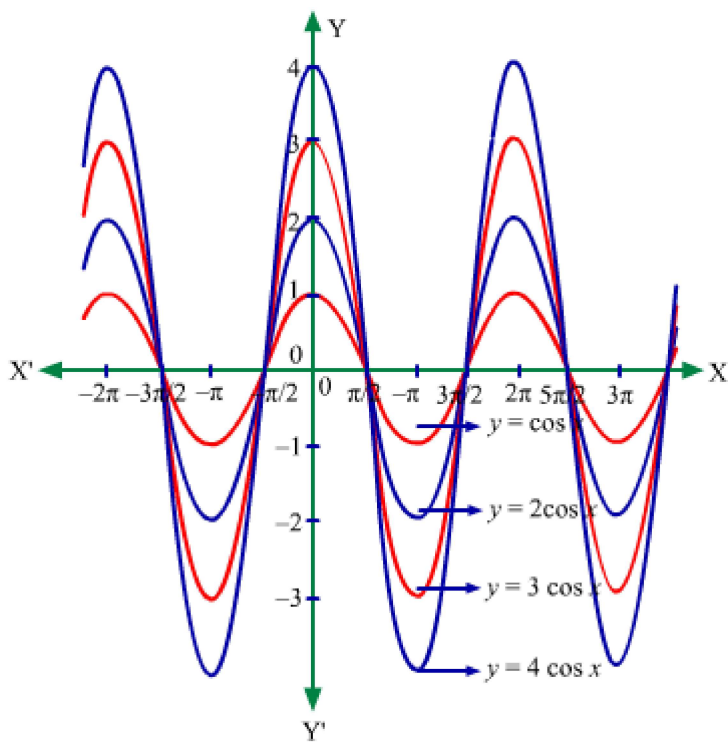


- Graphs of functions of the type $y = a \sin bx$:

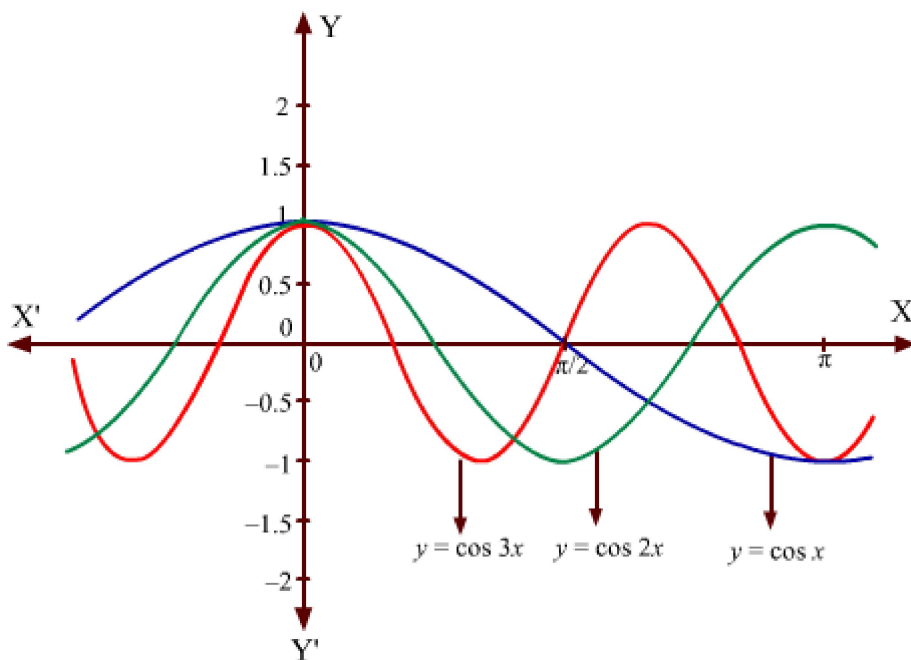
It is easy to understand such a function with the help of an example. Consider the function $y = 3 \sin 2x$.



- Graphs of functions of the type $y = a \cos x$:



- Graphs of functions of the type $y = \cos bx$:



- Graph of functions of the type $y = a \cos bx$:

It is easy to understand such a function with the help of an example. Consider the function $y = 3 \cos 2x$.

